Week 8 - Friday

COMP 4500

Last time

- What did we talk about last time?
- Integer multiplication
- Started Master Theorem

Questions?

Logical warmup

- A man works on the 10th floor of a building
- He always takes the elevator straight from the 10th floor to the ground floor at the end of the day
- Most mornings, he takes the elevator to the 7th floor and then walks the remaining three flights of stairs up to the 10th floor, even when in a hurry
- However, he takes the elevator straight to the 10th floor on mornings when others are in the elevator or when it is raining
- Why?

Master Theorem

Master Theorem

- Has a great name …
- Allows us to determine the Big Theta running time of many recursive functions that would otherwise take more effort to determine

Basic form the recurrence relation must take

$$T(n) = aT\left(\frac{n}{b}\right) + f(n),$$

$$\text{where } a \geq 1 \text{ and } b > 1$$

- a is the number of recursive calls made
- b is how much the quantity of data is divided by each recursive call
- $f(n)$ is the non-recursive work done at each step

Case 1

- If $f(n)$ is $\mathrm{O}\left(n^{\log_b(a)-\epsilon}\right)$
  for some constant $\epsilon > 0$, then

$$T(n) \text{ is } \Theta\left(n^{\log_b(a)}\right)$$

Case 2

- If $f(n)$ is $\Theta\left(n^{\log_b(a)} \log^k n\right)$ for some constant $k \geq 0$, then

$$T(n) \text{ is } \Theta\left(n^{\log_b(a)} \log^{k+1} n\right)$$

Case 3

- If $f(n)$ is $\Omega\left(n^{\log_b(a)+\epsilon}\right)$
for some constant $\epsilon > 0$, and if
$$af\left(\frac{n}{b}\right) \leq cf(n)$$
for some constant $c < 1$ and sufficiently large n,
then
$$T(n) \text{ is } \Theta(f(n))$$

Binary Search

- We know that binary search takes $O(\log n)$ time
- Can we use the Master Theorem to check that?
- What about merge sort?

Practicing the Master Theorem

- One way to practice is to try to create a problems that different cases of the Master Theorem apply to
- Give a recurrence relation that uses Case 1
- Give a recurrence relation that uses Case 2
- Give a recurrence relation that uses Case 3

Solved Exercises

Finding the maximum of unimodal data

- Imagine that array A contains unimodal data:
  - Values in A increase with index until they reach a maximum point
  - Then they decrease with index
- How can you efficiently find that maximum point?
- How long does your algorithm take to run?

Maximizing stock returns

- Consider historical stock prices over n days
- If you had a time machine and could go back and buy stock on day i and sell stock on day j (where $i < j$), which days would you pick to maximize your profits?
  - The goal is to buy as low as possible and to sell as high as possible
- First give an algorithm that runs in $O(n^2)$ time
- Now give one that runs faster
  - Hint: use divide and conquer

Upcoming

Next time…

- Review for Exam 2

Reminders

- **Finish Assignment 4**
  - **Due Monday**
- Exam 2 is next Wednesday
  - Review Chapters 4 and 5